Notes on the Concave Grating, and its Application to Stellar Photography

BY

S. ALFRED MITCHELL

A DISSERTATION SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS UNIVERSITY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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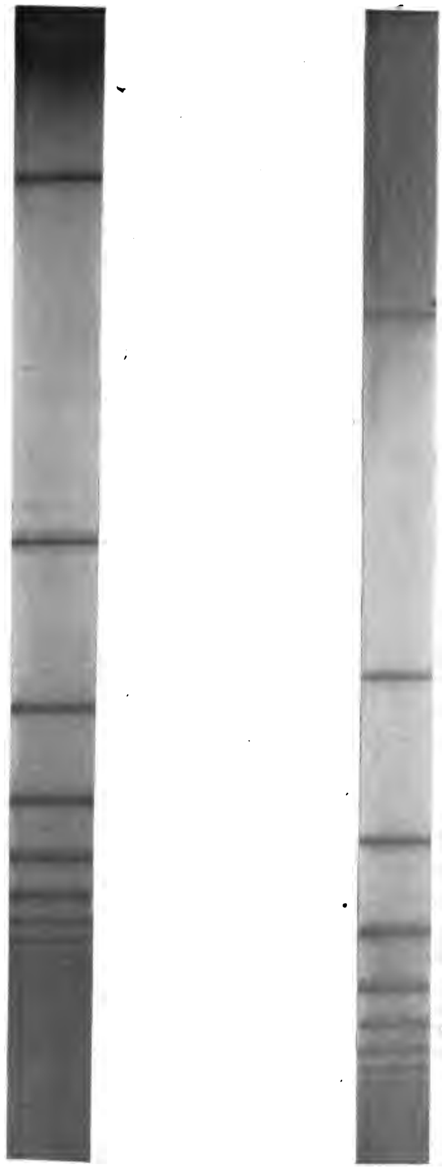

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NOTES ON THE CONCAVE GRATING.

By S. A. MITCHELL.

I. FUNDAMENTAL FORMULA.

THE general theory of the concave spherical grating has been investigated by Rowland (*Phil. Mag.*, **16**, 1883; and *Amer. Jour. Sci.* (3), **26**, 1883); by Glazebrook (*Phil. Mag.*, **15**, 1883); by Mascart (*Jour. de Phys.* (2), **2**, 1883); by Baily (*Phys. Soc. Proc.*, **5**, 1883); and by Kayser (*Winkelmann, Handbuch der Physik*, p. 408).

The following treatment is one which starts from a general condition, true for every form of grating ruled with equal spaces along a chord. By introducing the condition that the form of the grating is the section of a sphere, the general equation of the concave grating is obtained.

From the general theory of gratings (Lord Rayleigh, "Wave Theory of Light," § 14, *Ency. Brit.*, p. 437; and Kayser, *Handbuch der Physik*), we know, if we have a grating ruled with equal spaces on any surface, that

$$\lambda = \frac{\omega}{N} (\sin \gamma + \sin \mu). \tag{1}$$

where ω is the grating space, N the order of the spectrum, γ and μ the angles which the incident and diffracted light make with the normal to the surface at any point, and λ is the wave-length.

That is, in Fig. 1, if L is the radiant point, and a cone of rays from L falling on the surface of the grating QOT at P is brought to a focus at L' by any means (if the grating is plane a lens will be necessary), then λ is the wave-length of this light.

The above equation may be written:

$$(\sin \gamma + \sin \mu) = \frac{N\lambda}{\omega} \tag{2}$$

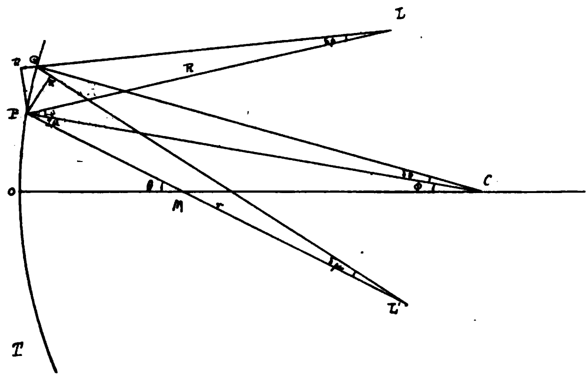

FIG. 1.

If now we allow the point P to move in the plane LoL', along the surface QOT, so that the angles which the incident and diffracted light make with the normal to the surface are now $(\gamma + \delta\gamma)$ and $(\mu + \delta\mu)$, then a new wave-length $(\lambda + \delta\lambda)$ will be brought to a focus at L', so as to satisfy the equation

$$\sin(\gamma + \delta\gamma) + \sin(\mu + \delta\mu) = \frac{N\lambda}{\omega} + \frac{\delta(N\lambda)}{\omega} \qquad (3)$$

Developing each term by Maclaurin's theorem, and subtracting (2), we get

$$\cos\gamma\, d\gamma - \tfrac{1}{2}\sin\gamma\,\delta\gamma^2 + \cos\mu\,\delta\mu - \tfrac{1}{2}\sin\mu\,\delta\mu^2 = \frac{1}{\omega}\delta(N\lambda) \qquad (4)$$

in which we neglect quantities of a higher order than the second. Hence, introducing an independent variable ϕ, we get:

$$\cos\gamma\frac{d\gamma}{d\phi} - \tfrac{1}{2}\sin\gamma\left(\frac{d\gamma}{d\phi}\right)^2 d\phi + \cos\mu\frac{d\mu}{d\phi} - \tfrac{1}{2}\sin\mu\left(\frac{d\mu}{d\phi}\right)^2 d\phi$$
$$= \frac{1}{\omega}\,\frac{d(N\lambda)}{d\phi}.$$

This equation gives the change in wave-length due to a change in the angles of incidence and diffraction.

If now the surface is spherical — the case of Rowland's concave grating — O is the "center of the grating," C the center of curvature, CP is normal to the surface at P, and the line CP is the radius of curvature ρ. Then the angle LPC is equal to γ,

$L'PC$ is equal to μ, and putting OCP' and OMP equal respectively to ϕ and θ, we see:

$$\gamma = \theta - \phi\,; \quad \mu = \theta' - \phi\,;$$

and hence,

$$\frac{d\gamma}{d\phi} = \frac{d\theta}{d\phi} - 1\,; \quad \frac{d\mu}{d\phi} = \frac{d\theta'}{d\phi} - 1 \tag{6}$$

Calling the distances LP and $L'P$ respectively R and r, we see that by letting fall perpendiculars PR and PR' on LQ and $L'Q$ respectively that

$$PR = \rho\, d\phi \cos\gamma = R\, d\theta.$$

Hence $$\frac{d\theta}{d\phi} = \frac{\rho\cos\gamma}{R}$$

and consequently $$\frac{d\gamma}{d\phi} = \frac{\rho\cos\gamma}{R} - 1\,.$$

Similarly, $$\frac{d\mu}{d\phi} = \frac{\rho\cos\mu}{r} - 1\,. \tag{7}$$

Substituting these values of $\frac{d\gamma}{d\phi}$ and $\frac{d\mu}{d\phi}$ in equation (5) we get:

$$\cos\gamma\left(\frac{\rho\cos\gamma}{R} - 1\right) + \cos\mu\left(\frac{\rho\cos\mu}{r} - 1\right) \tag{8}$$

$$= \frac{1}{\omega}\frac{d(N\lambda)}{d\phi} + \frac{1}{2}\,d\phi\left\{\sin\gamma\left(\frac{\rho\cos\gamma}{R} - 1\right)^2 + \sin\mu\left(\frac{\rho\cos\mu}{r} - 1\right)^2\right\}$$

which equation is true to terms of the order $d\phi^2$. To have a perfect focus for waves of length λ at L', this change in wave-length due to a change in the angle ϕ must be zero, or in other words $\frac{d\lambda}{d\phi} = 0$. Light of wave-lengths which are whole multiples of λ will also be brought to the same focus at L', since

$$\frac{d(2\lambda)}{d\phi} = 0, \quad \frac{d(3\lambda)}{d\phi} = 0 \quad . \quad . \quad \text{etc.}$$

Hence making $\frac{d(N\lambda)}{d\phi} = 0$, and neglecting infinitely small quantities of the first order $(d\phi)$, we get:

$$\cos\gamma\left(\frac{\rho\cos\gamma}{R} - 1\right) + \cos\mu\left(\frac{\rho\cos\mu}{r} - 1\right) = 0 \tag{9}$$

It may be noted that the omitted term contains as a factor:

$$\sin\gamma\left(\frac{\rho\cos\gamma}{R}-1\right)^2+\sin\mu\left(\frac{\rho\cos\mu}{r}-1\right)^2. \tag{10}$$

Equation (9) may be put in the form

$$\frac{\cos^2\gamma}{R}+\frac{\cos^2\mu}{r}=\frac{\cos\gamma+\cos\mu}{\rho}, \tag{11}$$

whence we get:

$$r=\frac{R\rho\cos^2\mu}{R(\cos\gamma+\cos\mu)-\rho\cos^2\gamma}. \tag{12}$$

This is the equation of the curve on which the spectra are brought to a focus. In this, the center of the grating is the origin, the line passing through the center of curvature is the axis of reference, R and γ the coördinates of the source of light, r and μ the coördinates of the spectral line.

This is the same equation as derived by Rowland (*loc. cit.*).

Further, we see that equation (11) is satisfied by making $R=\rho\cos\gamma$, $r=\rho\cos\mu$. The same substitution makes the omitted term, viz. (10), vanish. Consequently, if the conditions $R=\rho\cos\gamma$, $r=\rho\cos\mu$ are satisfied, formula (12) is true to terms of the second order. (See Kayser, *loc. cit.*) This condition is secured automatically in "Rowland's Mounting," where $R=\rho\cos\gamma$, $\mu=0$. Therefore $r=\rho$. (See Ames, *Johns Hopkins Circulars*, May, 1889.)

This mounting consists in having slit, grating, and camera at the vertices of a right angled triangle, the camera being placed at the center of curvature of the grating.

II. ASTIGMATISM.

One of the most important properties of a concave grating is its "astigmatism," *i. e.*, the fact that a point of light as a source gives rise to a focus, not a point, but a line. The advantages arising from this fact, as pointed out by Ames, (*Johns Hopkins Circulars*, May 1889) are:

1. A narrow spark at the slit is broadened out into a wide spectrum

2. Greater accuracy in comparing metallic and solar lines.
3. No "dust lines," as they are brought to a different focus
4. A spectrum is obtained which is broad enough to stand enlarging.

Although this property has always been recognized, no formula giving the amount of the astigmatism has ever been published, at least to my knowledge. This quantity is therefore deduced in the following pages.

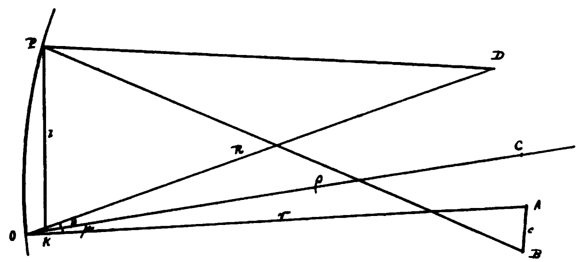

FIG. 2.

In Fig. 2 let D be the position of the point of light at the slit situated at a distance R from the center of the grating O; AB is the spectral line under consideration, which is consequently parallel to the lines of the grating, and is situated at a distance r from the center of the grating; C is the center of curvature; CO is the radius of curvature ρ. A, O, C, and D lie in a horizontal plane.

Let PO be a vertical section of the grating, *i. e.*, parallel to the lines of the grating. A small pencil of rays from the slit D falling on the grating at O is brought to a focus at A; a small pencil from D falling on the grating at P is brought to a focus at B. It is our problem to find the length of AB, assuming AB to be perpendicular to OA.

Call $AB = C$, $PK = Z$, where PK is the perpendicular let fall from P on the radius of curvature OC.

Since OK is a very small quantity compared with the other quantities considered :

$$OK = \frac{Z^2}{2\rho}, \text{ and}$$

$$AK = r - \frac{Z^2}{2\rho}\cos\mu.$$

$$BP^2 = AK^2 + (KP + AB)^2.$$

$$= \left(r - \frac{Z^2}{2\rho}\cos\mu\right)^2 + (Z + C)^2.$$

If we neglect terms of a higher order than the second we shall have:

$$BP = r - \frac{Z^2}{2\rho}\cos\mu + \frac{Z^2 + 2CZ + C^2}{2r}.$$

Similarly we shall have:

$$DP = R - \frac{Z^2}{2\rho}\cos\gamma + \frac{Z^2}{2R}.$$

But since the point of light at D and the focal line AB are true foci, the differences in paths of rays falling at P and at O must be zero. Hence

$$(AO + DO) - (BP + DP) = 0$$

$$i.e. \frac{Z^2}{2}\left(\frac{\cos\mu}{\rho} + \frac{\cos\gamma}{\rho} - \frac{1}{R} - \frac{1}{r}\right) - \frac{CZ}{r} - \frac{C^2}{2r} = 0. \quad (13)$$

Solving we get:

$$C = -Z \pm Z\sqrt{r\left(\frac{\cos\mu + \cos\gamma}{\rho} - \frac{1}{R}\right)}. \quad (14)$$

Twice this value will give us the length of the line, which is parallel to the lines of the grating, when we have a point-source of light at the slit. But in practice the illuminated portion of the slit, placed parallel to the lines of the grating, is of some finite length $2b$, where b is the length of the illuminated portion of the slit above the horizontal plane. To find the length (c') of the lines of the spectrum due to this, we have merely to apply the ordinary equation for finding the magnification due to a concave mirror:

$$\frac{C'}{r} + \frac{b}{R} = 0$$

$$\text{or } C' = -\frac{br}{R}$$

Adding this to the value from equation (14) we find half the length of the spectral lines :

$$C_0 = -Z \pm Z\sqrt{r\left(\frac{\cos\mu + \cos\gamma}{\rho} - \frac{1}{R}\right) - \frac{br}{R}}.$$

Where Rowland's mounting is used the photographic plate is at the center of curvature, and hence

$$r = \rho,$$
$$\mu = 0,$$
$$R = \rho\cos\gamma.$$

Substituting these values in the above equation we get

$$C_0 = -\frac{b}{\cos\gamma} - Z \pm Z\sqrt{1 - \frac{\sin^2\gamma}{\cos\gamma}}.$$

Taking the special case when $\gamma = 0$, the image of the slit coincides with the slit itself, and hence $C_0 = -b$. Whence we see that the positive sign before the radical must be used. Hence

$$C_0 = -\frac{b}{\cos\gamma} - Z + Z\sqrt{1 - \frac{\sin^2\gamma}{\cos\gamma}}. \qquad (15)$$

From this equation it is seen that the astigmatism depends only on the length of the slit, the length of the ruled lines of the grating, and the angle which the source of light makes with the axis of the grating. Astigmatism is *independent* of the radius of curvature.

In this formula C_0 is half the length of the spectral line, b is half the length of the illuminated portion of the slit, Z is half the length of the ruled lines of the grating. The angle γ is computed from the wave-length by the formula:

$\lambda = \frac{\omega}{N}(\sin\gamma + \sin\mu)$, which for the case of Rowland's mounting, where $\mu = 0$, reduces to

$$\lambda = \frac{\omega}{N}\sin\gamma.$$

Measuring the lengths of the lines of the spectrum formed by the large concave grating spectroscope used in the Johns

Hopkins University, we get the following table, giving the lengths measured and those computed from the above formula. The width of the spectrum was measured by holding a scale in front of the camera box and observing by means of an eyepiece held in the hand.

The grating has a radius of curvature of $21^{ft}.5$, is ruled with 20,000 lines to the inch, and has a ruled surface of 2^{in} by $5^{in}.66$. The length of the slit used was 3^{mm}. In the following table the first column gives the line of the spectrum, the second the wave-length (λ), the third the order of spectrum (N), the fourth the value of γ, the fifth the computed length of the spectrum line λ, and the sixth its observed length.

Line	λ	N	γ	Computed	Observed
h	4102	1	18° 50′ 38″	6 mm	6mm
G	4308	1	19 49 44	6.5	7
F	4861.5	1	22 30 25	8	9
b_1	5184	1	24 5 28	8.5	11
D	5893	1	27 38 47	10	15
C	6563	1	31 6 57	12.5	17
B	6767	1	32 11 50	13	19
h	4102	2	40 14 22	21	29
G	4308	2	42 43 14	24	32
F	4861.5	2	49 57 34	46	51

Special case.—When the concave grating is used directly as a star spectroscope, the source of light is at infinity, and the slit-length is zero. In this case the fundamental equation (12) reduces to

$$r = \frac{\rho \cos^2 \mu}{\cos \gamma + \cos \mu} \cdot$$

Hence, making $R = \infty$, and giving r this value, equation (14) reduces to

$$C = -2 Z \sin^2 \frac{\mu}{2} \cdot$$

The grating is used as a star spectroscope in two ways, either the photographic plate is in the axis of the grating, when $\mu = 0$ for the center of the plate, or the photographic plate is

placed so as to make $\gamma = 0$, *i. e.*, the light falls normally on the grating.

In the present work, with the direct grating spectroscope (see Poor and Mitchell, "The Concave Grating for Stellar Photography," this JOURNAL, March 1898), the large grating used had a ruled surface of 2^{in} by $5\frac{3}{4}^{in}$. The whole spectrum in the second order subtended an angle of about 6°. The grating was used in such a way that $\mu = 0$, at the center of the plate; and consequently for a variation of 3° in μ, the astigmatism amounted at most to $0^{in}.003$.

If the light (in the other method of using the grating) falls normally on the grating, *i. e.*, if $\gamma = 0$, then μ is about equal to 15°, and so the astigmatism amounts to about $0^{in}.07$.

Hence the grating used directly as a star spectroscope gives a spectrum which is exceedingly narrow.

III. FOCAL LINES.

By treating equation (13)

$$\frac{Z^2}{2}\left(\frac{\cos\gamma + \cos\mu}{\rho} - \frac{1}{R} - \frac{1}{r}\right) - C\frac{Z}{r} - \frac{C^2}{2r} = 0$$

in a slightly different way, some interesting results are obtained. The equation gives the position of the focus conjugate to a point source; C will be zero for either of two conditions:

$$Z = 0$$

$$\text{or} \quad \frac{\cos\gamma + \cos\mu}{\rho} - \frac{1}{R} - \frac{1}{r} = 0 \qquad (16)$$

That is, under either of these conditions, the focus is a point, not a line.

The meaning of equation (16) will be better understood if we regard the grating from another standpoint. Using the grating in the ordinary way, a cone of rays falls obliquely on its surface, and is diffracted and brought to a focus in a spectral line.

Thus we may consider the grating as some sort of a mechanism which changes a pencil of rays falling obliquely on the grat-

ing into a pencil leaving at some different angle. This action may be considered in two ways, according as we take a meridian section of the grating, *i. e.*, a section passing through the two focal points and the center of the grating; or, as we take an equatorial section, *i. e.*, a section of the grating at right angles to this. The equations of oblique pencils refracted at a spherical surface of a medium whose index of refraction is p have been fully treated and discussed by Czapski (Winkelmann, *Handbuch der Physik*, **2**, 85, 86); and by Rayleigh (*Ency. Brit.*, **14**, 800, art. "Optics").

These equations give the position of the *focal lines* for a meridian section and for an equatorial section. They are:

$$\frac{\cos^2\gamma}{R} - \frac{p\cos^2\mu}{r} = \frac{\cos\gamma - p\cos\mu}{\rho}, \tag{17}$$

$$\frac{1}{R'} - \frac{p}{r'} = \frac{\cos\gamma - p\cos\mu}{\rho}, \tag{18}$$

where R and R' are the distances of the radiant point from a point in the meridian and equatorial sections respectively, r and r' are the distances of the focal lines from the same point of the surface, ρ is the radius of curvature, γ and μ are the angles which the incident and refracted rays make with the normal at the surface.

Applying this to the case of the concave grating by putting $p = -1$, we get

$$\frac{\cos^2\gamma}{R} + \frac{\cos^2\mu}{r} = \frac{\cos\gamma + \cos\mu}{\rho}, \tag{19}$$

$$\frac{1}{R'} + \frac{1}{r'} = \frac{\cos\gamma + \cos\mu}{\rho}. \tag{20}$$

Equation (19) fixes the positions of the radiant and focal lines for a meridian section; equation (20), the position for an equatorial section. As the grating is generally used, the meridian section becomes a horizontal one, the equatorial section a vertical one. Thus equation (19) means that, if we use a horizontal section of the grating—a section perpendicular to the lines of the grating—we get the position of the conjugate foci;

a vertical line at the slit with coördinates R and γ is brought to a focus on the photographic plate in a vertical line whose coördinates are r and μ. This is the same equation as Rowland's. Equation (20) (which is identical with equation (16) gives the position of the radiant and focal lines using a vertical section of the grating, *i. e.*, a section parallel to the lines of the grating. This equation is satisfied by making $r' = \rho \sec \mu$, $R' = \rho \sec \gamma$; or making $\mu = 0$ (as in Rowland's mounting) by $r' = \rho$, $R' = \rho \sec \gamma$. From the theory of these focal lines (see *Ency. Brit.*, *loc. cit.*) we see that corresponding to a point-source this focal line is horizontal, and conversely. Consequently, using a horizontal slit at a distance $\rho \sec \gamma$, and the angle γ from the grating, light from it, after passing through the vertical slit, will be brought to a focus in a point in the spectrum. This fact is shown by placing the source of light at some distance in front of the slit (*i. e.*, away from the grating), and by placing an obstacle, for example a knitting needle, horizontally in front of the vertical slit and distant $\rho \sec \gamma$ from the grating. Doing this, we shall get a sharply defined horizontal line running across the spectrum. (See Sirks, *Astronomy and Astro-Physics*, **13**, 1894.)

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THE CONCAVE GRATING AS A STAR SPECTROSCOPE.

THERE are two radically different methods of using the concave grating as a star spectroscope. In the first of these methods the concave grating is used to replace the ordinary spectroscope at the eye end of the telescope, the whole forming a compound grating spectroscope. The second method is where the light from the star is allowed to fall directly on the grating, from which it is diffracted and brought to a focus on the photographic plate.

Both methods were originally suggested by Professor Rowland immediately after his invention of the concave grating in 1882.

COMPOUND GRATING SPECTROSCOPE.

The first method of using the concave grating for stellar photography is one where we apply to the equatorial Rowland's usual manner of mounting slit, grating, and photographic plate at the vertices of a right-angled triangle.

This method was tried in 1892 by Professor Henry Crew at the Lick Observatory (*Astronomy and Astro-Physics*, XII, 1893). He obtained a few results. His "mounting was a wooden one designed and made on the spot," and was therefore hardly built carefully enough to thoroughly test the usefulness of the grating as a star spectroscope. It was to test this more thoroughly that a careful mounting was constructed which could be attached to the equatorial belonging to this university.

This telescope is a $9\frac{1}{2}^{\mathrm{in}}$ visual one, the object glass of which was designed and polished by Professor C. S. Hastings while he was connected with this university; the mounting is by Warner & Swasey.

The grating used has a radius of curvature of 1^{m} with a ruled surface of 1×2 inches, and ruled with 15,000 lines to the

inch. The support for the grating was one made of three half-inch hollow steel tubes, forty-eight inches long, the ends near the grating being fastened to a circular brass plate five inches in diameter, the other ends being fastened to a ring thir-

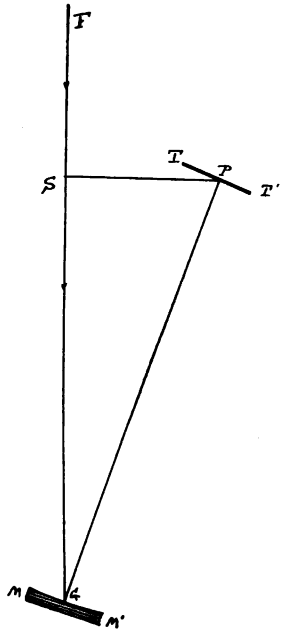

FIG. 3.

teen inches in diameter, which could be tightly clamped against a heavy ring of similar size that is rigidly fastened to the eye end of the telescope. The whole made a rigid mounting in which little, if any, displacement was possible. To two of the steel rods supporting the grating was brazed another short rod which was in the focal plane of the telescope, and this rod, which supported the camera box, also served to make the whole mounting more rigid. The whole mounting was covered with black cloth, and altogether constituted a camera.

The grating was attached to a holder which had the ordinary side and back slow-motion screws, and this holder in turn was

attached to a short rod which passed centrally through the outer brass plate of the mounting.

The camera consisted of a light wooden box, which held a small plate holder in which a plate 1×5 inches could be bent as nearly as possible to the proper radius of curvature. The first spectrum alone was used, and with this the length of the photographic spectrum was about $3\frac{1}{2}$ inches. Consequently, the whole spectrum could be photographed on one plate, and it was not necessary to have the camera on a carriage which is capable of moving, as in Rowland's usual mounting.

In Fig. 3, the light from the star comes in the direction $F\ S\ G$, the axis of collimation of the telescope. After being focused on the slit at S, the light falls on the grating $M\ G\ M'$, and is diffracted to the photographic plate at $T\ P\ T'$.

Adjustments of the apparatus.—The camera-box could be moved along $S\ P$, along $P\ G$, and along an axis through P perpendicular to $S\ P$ and $P\ G$; and it could also be rotated about this latter axis.

The grating was capable of being moved along the line $G\ F$, of being rotated about this line as an axis, and also about an axis passing through G perpendicular to $S\ G$ and $P\ G$. The side and back screws of the grating gave these two latter motions as slow motions. Camera-box and grating were joined together by means of a steel tube of the same diameter as the others used; and by means of clamp screws each could be moved separately, or they both could be moved together.

The slit S was attached to the focusing apparatus of the telescope, and so it could be moved along the axis of collimation $F\ G$, and could also be rotated about this axis.

The distance $P\ G$ must equal the radius of curvature of the grating, and the grating and plate—in order to get a normal spectrum—must each be perpendicular to $P\ G$. These and the other necessary adjustments of the apparatus are made in exactly the same manner as in the usual Rowland mounting (see Ames, "Grating in Theory and Practice," *Johns Hopkins Circulars*, May 1889, *Astronomy and Astro-Physics*, XI, 1892).

To use this grating as a star spectroscope, the slit is placed

in the photographic focus of the telescope with the length of the slit so as to follow in right ascension. In this way, the slight irregularities of the driving-clock do not make the image of the star move off the slit. The lines of the grating are placed so as to be parallel to the slit. To find the photographic focus, a photographic plate may be clamped close to the slit, and directing the telescope to some bright star, a series of equal exposures may be made with the slit at different positions along the axis of collimation, each time shifting the plate a little. On developing the plate, the position of maximum sensibility, which is readily seen from the images on the plate, gives the photographic focus of the object glass.

While photographing the spectrum of a star it is necessary to keep the image of the star constantly on the slit. This is usually one of the most serious obstacles to be overcome in the use of any star spectroscope, but by the use of the grating this is accomplished very easily. On a line inclined to the axis of the grating at the same angle as the slit, the light from the slit is brought to focus in two focal lines. Observing the vertical line, *i. e.*, the line parallel to the slit, we see diffraction bands caused by the slit, and also bright and dark portions of the field. Thus, by placing an eyepiece on an arm attached to the mounting so as to observe this focal line, and also placing cross-hairs in the focal plane of this eyepiece, the cross-hairs can be kept at a definite position by means of the slow-motion screws of the telescope. Accordingly, the image of the star may be kept from moving off the slit, not only in declination, but also in right ascension.

Method of use.—The adjustments were all made in daylight in the manner already described, and narrowing the slit as much as possible, a photograph of the Sun's spectrum was taken. This was repeated at different foci, and the position giving the best definition, as seen from the photographs, was that used for photographing stars. The slit was widened for star work, and for my experiments, Sirius was the star observed.

The results of the photographs of Sirius give six of the

hydrogen lines, and many other fine lines, including K. The definition is about equal to that obtained by the use of prisms.

Compared with a prism spectroscope, the compound concave-grating spectroscope has the following advantages when used for stellar observations.

1. A star can be followed very easily.

2. The grating is astigmatic, and thus it is not necessary to widen out the spectrum.

3. The spectrum is "normal," and thus wave-lengths can be directly measured.

With these marked advantages in favor of the use of the grating, it would need something of importance on the other side to offset them; and this is found in the long exposure which must be given. On Sirius, I was surprised to find the length of exposure between three and four hours. Of course, the telescope used was a visual one, not a photographic one, as it should have been, and so the image on the slit is not a point of light. But even with the use of a photographic telescope, the time of exposure could probably not be made any less than three hours; and so the grating used to replace the ordinary prism-spectroscope at the eye end of the telescope will not become of much practical value.

The results of some of the individual plates taken with the grating used in this way are given in the annexed table:

Plate	Date	Star	Exposure	Width	Remarks
58	Feb. 6	Sirius	120^m	6^{mm}	6 Hydrogen bands K. Some fine lines.
67	" 12	"	120	7	5 " " K. 15 other lines.
68	" 13	"	100	6	6 " " K. Some lines.
70	" 16	"	240	9	7 " " K. 50 other lines.
77	" 27	"	240	4	5 " " K. Some other lines.
81	" 28	"	75	2	5 " " K. Some underexposed.
82	Mar. 1	"	240	2	7 " " K.
83	" 5	"	240	2	5 " " K.
84	" 7	"	240	2	5 " " K.

Plates 58–77 were taken by allowing the star to trail somewhat in right ascension. These plates are not as dense as plates 82, 83, and 84, but they all show the lines quite clearly.

The whole spectrum is about 8.5^{cm} long, the distance from H_β to H_γ about 3^{cm}.

On account of this great length of exposure, and the continual jarring to which the whole Observatory is subjected, the definition is somewhat poor in the resulting photographs. But photographs and visual observations of the Sun's spectrum give definition much better than that obtained from the photograph of Sirius; and it seems to compare very favorably with the definition obtained by other methods. It is in fact just as good as could be expected by the use of a visual telescope for photographic work.

In summing up, we can say that this form of spectroscope has many advantages in its favor, but these advantages are outweighed by the long exposure which is necessary.

DIRECT GRATING SPECTROSCOPE.

In the second method of using the concave grating as a star spectroscope, the light from the star falls directly on the grating, is diffracted, and brought to a focus on the photographic plate. This method, originally suggested by Professor Rowland, was in 1891 taken up by Dr. Charles Lane Poor, who had a rough apparatus made and obtained a few results. At his suggestion, the work was continued by Mr. S. V. Hoffman, of this university. He obtained some photographs of Sirius and Vega, but for various reasons the work was never completed.

As far as I know, these were the only experiments made until the work was again taken up in October 1897.

In the *Astrophysical Journal* of January 1896, is an article by Professor F. L. O. Wadsworth, in which this method is treated and the equations given. Professor Wadsworth, however, finds that the spectrum is not "normal," a conclusion which is not quite correct.

Since the star is a point of light, a slit is not necessary; and thus the greatest difficulty in the way of procuring a spectrum photographically—that of keeping the image of the star accurately on the slit—is overcome.

Since the light passes through neither objective nor prism trains, there is no absorption of the light, and thus in photographic work the light extends far into the ultra-violet.

It is also found that, used in a certain position, the grating gives a "normal" spectrum, thus affording a means of readily and quickly measuring wave-lengths.

These are indeed very great advantages in favor of using this form of a spectroscope, and unlike the method of using the grating at the eye end of the telescope, the length of exposure necessary is very short. On Sirius, using the first order of spectrum, with the 6-inch grating, enough is obtained on the plate in three minutes to show every detail.

By getting two spectra on the same plate—as will soon be shown—measures may be obtained of the absolute wave-lengths, and thus the motion in the line of sight can be determined.

Thus the advantages of using this form of spectroscope in preference to any other form at present in use are apparent.

From the theory of the concave grating we have the general equation of the curve on which the spectrum is in focus:

$$r = \frac{R \rho \cos^2 \mu}{R (\cos \mu + \cos \gamma) - \rho \cos^2 \gamma}. \tag{23}$$

In this equation ρ is the radius of curvature of the grating the axis of the grating is the line of reference for angular measurements, and the center of the grating the origin of polar coördinates; R and γ are the coördinates of the source of light; r and μ those of the curve on which the spectra are brought to a focus.

This equation may be put in the following form:

$$r = \frac{\rho \cos^2 \mu}{\cos \mu + \cos \gamma - \frac{\rho}{R} \cos^2 \gamma}. \tag{24}$$

If now the source of light be placed at an infinite distance, R is equal to infinity, and the equation (24) reduces to:

$$r = \frac{\rho \cos^2 \mu}{\cos \mu + \cos \gamma}. \tag{25}$$

We may use the grating in a number of different ways depending on the position of the photographic plate and of the source of light in reference to the axis of the grating, *i. e.*, depending on the different values of μ and γ in the above equation. Since this equation gives the curve on which the spectral lines are brought to a focus, the photographic plate must be bent in conformance with this curve.

One way of using the grating is when the center of the photographic plate is on the axis of the grating. For this case $\mu = 0$, and our general equation of the curve of focus reduces to:

$$r = \frac{\rho}{1 + \cos \gamma} . \tag{26}$$

For any fixed value of γ, r is constant, and those parts of the spectra, for which we can assume $\cos \mu$ equal unity, are brought to a focus on a circle whose radius is r as given in (26). When γ varies this equation is that of a parabola. Hence when γ is changed (μ having kept equal zero) different orders of spectra are brought to a focus on a parabola as given by (26). This case is shown in the following figure:

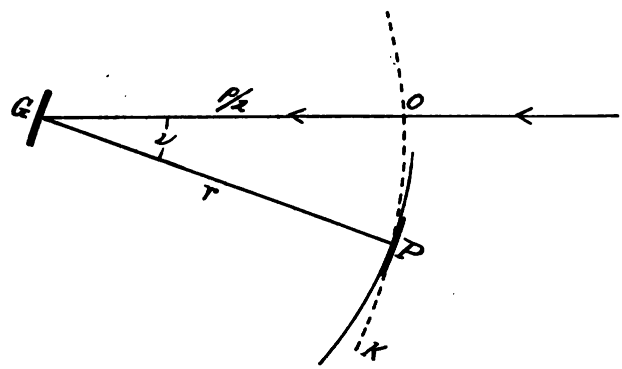

FIG. 4.

G is the grating, *P* the photographic plate. The light comes from the star in the direction *O G*, falls on the grating, and is diffracted to the photographic plate *P*. The curve *OPK* is the parabola, *OG* being the half parameter and equal $\frac{1}{2} \rho$. For a

constant value of γ, those spectral lines on the photographic plate, for which $\cos\mu$ can be assumed equal to unity, are brought to a focus on a circle whose radius is r,

Equation (25) may also be written:

$$r = \frac{\rho}{1 + \dfrac{\cos\gamma}{\cos\mu}} \cdot \cos\mu \tag{27}$$

and if μ and γ differ very little from each other, this will take the form:

$$r = \frac{\rho}{2}\cos\mu \,. \tag{28}$$

This is the equation of a circle with $\rho/2$ as diameter. In the present work, μ and γ are small, and thus this approximate equation gives the curve to which a plate must be bent. The photographic plate must therefore be bent to a radius of $\rho/4$.

To investigate this case fully, we must return to the general equation (25).

For a fixed value of γ, all the spectra are brought to a focus on the curve:

$$r = \frac{\rho\cos^2\mu}{\cos\mu + \cos\gamma}$$

in which $\cos\gamma$ is a constant. When μ is so small that its cosine may be taken as unity, this curve reduces to a circle as above described.

By the general theory of gratings we have:

$$\lambda = \frac{\omega}{N}(\sin\mu + \sin\gamma) \tag{29}$$

where ω is the grating space, and N the order of the spectrum. From this we have at once, γ being constant

$$\frac{d\lambda}{d\mu} = \frac{\omega}{N}\cos\mu \,. \tag{30}$$

To find the change in wave-length as we pass along the focal curve we have:

$$\frac{d\lambda}{ds} = \frac{d\lambda}{d\mu} \cdot \frac{d\mu}{ds} \,.$$

and

$$\frac{d\mu}{ds}=\frac{1}{\sqrt{r^2+\left(\frac{dr}{d\mu}\right)^2}}.$$

Differentiating the equation of the focal curve we have:

$$\frac{dr}{d\mu}=\frac{r\cos\mu-\rho\sin 2\mu}{\cos\mu+\cos\gamma}=\phi(\mu) \tag{31}$$

Whence substituting we finally find:

$$\frac{d\lambda}{ds}=\frac{\omega}{N}\cdot\frac{\cos\mu}{\sqrt{r^2+[\phi(\mu)]^2}} \tag{32}$$

This is the general formula for change in wave-length along the focal curve.

If now we put $\mu=0$, this reduces to

$$\frac{d\lambda}{ds}=\frac{\omega}{N}\cdot\frac{1}{r_0} \tag{33}$$

a constant. Hence at this point the spectrum is "normal." Within the limits, therefore to which we can take $\cos\gamma$ as equal to unity, the spectrum may be considered as normal.

For a grating of moderate dispersion the entire spectrum will be practically normal, provided the center of the plate is on the axis of the grating, *i. e.*, μ equal zero for the middle of the spectrum. In the small grating actually used the entire first order spectrum subtended an angle of 6°; and by computation it is found that from the above formula (32) the scales of different portions of the spectrum differ by less than three parts in a thousand. To be more exact, at a point 3° from the axis, the scale is smaller than at the center of the plate, the ratio of the latter to the former being 1.0025. On a plate of the solar spectrum as taken by the usual twenty-one foot Rowland mounting, the scales of the end and center differ in the ratio 1.0015 for the same variation of 3°.

Thus from the mathematical discussion of the subject, we see that to get the spectral lines in focus we must bend the photographic plate in conformance with the curve (25), which we see is very approximately satisfied by bending the plate to a radius

of $\rho/4$. In order to get a normal spectrum, the grating and plate must be placed so that they are each perpendicular to the line joining them.[1]

In order to test the above method, a small Rowland concave grating with a ruled surface of 1×2 inches was used. The grating has a radius of curvature of one meter, and is ruled with 15,000 lines to the inch. The apparatus for mounting the grating is extremely simple, consisting of a light box clasped to the tube of the equatorial, the telescope being used merely as a finder.

The light from the star falls directly on the grating, is diffracted and brought to a focus on the photographic plate. The grating is mounted in an ordinary holder which is capable of being adjusted by means of side and back screws, and which can also be rotated about an axis passing through the center of the grating and parallel to the lines of the grating. The holder, which has a circular base, is fastened to a concentric plate of slightly larger diameter on which is a scale divided to degrees, by means of which the amount of the above rotation can be read off.

The plate holder holds a plate 1×5 inches, bent as closely as possible to the proper radius. The holder is capable of adjustments along the line joining grating and plate, along the line perpendicular to this latter direction, and also of a rotation perpendicular to these two directions. The first of these is the adjustment used for focusing, and this is done by means of a screw. Scales are attached to each of these, and the amount of shifting or turning is read off directly.

By these adjustments the plate and grating are made parallel in order to procure a normal spectrum, and the plate is put in the best possible focus.

The box is clamped to the telescope in such a way that the lines of the grating are parallel to the equator, and accordingly, by regulating the driving clock of the telescope to run a little too slow or too fast, the spectrum can be made of any convenient

[1] The above formula was deduced by Dr. Poor (See *Astrophysical Journal*, March 1898).

width. This can be done by placing the telescope in the meridian, and clamping the box in the right position by means of a plumb line. Using the grating in this way the spectrum of a star, as we have seen, is practically a line of light which for photographic work may be widened out in the manner already stated.

Used on the Sun, the grating gives a spectrum of some width depending on the focal length of the grating, and the edge of this spectrum will be perfectly sharp when the plate is in focus. In this way the photographic plate is approximately put in focus, and turned so as to be approximately parallel to the grating, which position is found by turning the plate so as to give a sharp spectrum throughout. These adjustments are made more carefully by measurements, making $\mu=0$ in equation (29) we get:

$$\lambda = \frac{\omega}{N} \sin \gamma .$$

Knowing ω, the grating space, and N, the order of the spectrum used, we can find the angle γ through which the grating must be turned for any definite wave-length. The photographic plate can also be placed in the correct position by measurements in accordance with equation (26). This latter adjustment for getting plate and grating parallel may be secured as in the ordinary mounting of the concave grating, namely, by holding a candle somewhat in front of the plate holder in the line joining grating and holder, and turning the grating until candle and its image coincide, the grating is then made perpendicular to the line joining grating and holder. By putting a piece of plane mirror on the back of the plate holder and holding the candle near the grating, and turning the plate holder until candle and its image coincide, the plate holder is then made perpendicular to the line joining holder and grating.

The manner in which a photograph is taken by this form of spectroscope is extremely simple. One side of the box near the plate holder is on hinges so that the plate holder can readily be put in position. During the exposure, the star is allowed to

trail somewhat in right ascension by regulating the clockwork to run a little too slow or too fast on sidereal time. The star is followed in the telescope, and by means of the filar micrometer the star is kept from moving in declination, and as we can take advantage of the high magnifying power of the telescope, the star can be kept accurately from moving in declination with reference to the grating. In this way, by keeping the star's image steady, the definition obtained is very good. As we have no slit on which the image of the star must be kept, and as the star can be followed so extremely accurately, the experimental difficulties in the way of obtaining a photographic spectrum are extremely slight.

For our trials Sirius was the star principally observed, and exposures ranged from ten minutes to one hour according to the width of the spectrum. All photographs were made with the first order spectrum, and Seed's Gilt Edge plates were used. The spectra are about 5^{cm} long and vary in width from 0.1^{mm} to $1^{mm}.5$ depending upon the exposure and the rate of the clock.

Details of a few of the plates taken by means of the small grating are given in the following table:

Plate	Date	Star	Expos.	Width	Remarks
	1897		min.	mm	
8	Nov. 27	Sirius	40	1 .5	8 Hydrogen bands, H and K.
9	Dec. 9	Capella	40	0 .2	F, G, h, H, K, and about 50 fine lines.
12	Dec. 12	Sirius	40	0 .3	10 Hydrogen bands, H and K, and 20 fine lines.
14	Dec. 15	Procyon	40	0 .1	6 Hydrogen bands, H and K, and 20 fine lines.
15	Dec. 15	Sirius	40	0. 3	13 Hydrogen bands, H and K.
16	Dec. 26	Sirius	40	0.15	13 Hydrogen bands, H and K.
21	Dec. 27	Rigel	85	0 .1	14 Hydrogen bands, H and K, and 6 other lines.
24	Dec. 28	Sirius	40	0.15	13 Hydrogen bands, H and K.
	1898				
27	Jan. 3	Sirius	46	0 .2	14 Hydrogen bands, H and K. 10 other lines.
29	Jan. 3	Sirius	40	0 .2	16 Hydrogen bands, H and K. 15 other lines.

The plates 9, 14, and 21 are underexposed.

From the character of the plates, and from the facts that we can easily see 16 hydrogen bands in Sirius (plate 29), and also so many fine lines in Capella (plate 9), we see that we get a great ultra-violet extension, and also that the definition is extremely fine. Taking into account the smallness of the ruled surface of the grating — a surface of only 1×2 inches — the length of exposure necessary is one which is not excessive.

Hence we can see the many advantages of this form of a spectroscope, advantages through the ease with which the spectrum is obtained, and advantages through the greater usefulness of the plate to determine the wave-lengths.

LARGE GRATING.

The results obtained by such a small grating — the ruled surface being only 1^{in} by 2^{in} — were so promising that a large grating was ruled. It has a ruled surface of 2^{in} by $5\frac{3}{4}^{in}$ with a radius of 1 meter, and is ruled with 7219 lines to the inch, the spectra on the two sides being about equally bright. It was mounted on the equatorial in the way already described.

The first and second order spectra on one side were taken on one plate which was bent as closely as possible to the desired radius of curvature. The plates were 1^{in} by 5^{in}, and Seed's Gilt Edge plates were used. On these plates the second order spectra had a length of about 5^{cm}, the distance from H_β to H_γ about $1^{cm}.5$.

Considering the situation of the Observatory, which is continually subject to the jars of the city traffic, the definition on the plates is splendid, much surpassing what could be expected.

The following table gives the results of a few of the plates taken with the grating, the exposure being correct for the second order of spectrum, the first in every case being overexposed. If plates could be obtained which could be bent to the proper radius — this is possible by using a grating of greater focal length — even better definition could be obtained in the ultra-

violet part of the spectrum. The last column shows the good definition obtained by the grating.

Plate	Date	Star	Expos.	Width	Remarks
	1898		min.	mm.	
97	March 15	Vega	27	0.30	14 Hydrogen bands, K.
108	" 31	"	20	0.35	14 " " K.
115	April 2	"	10	0.20	14 " " K.
119	" 3	Sirius	10	0.35	13 " " K.
121	" 4	Procyon	20	0.35	11 " " K. Many fine lines.
122	" 4	Capella	20	0.35	F, G, h, H, K, and many fine lines.
124	" 5	Procyon	30	0.40	11 Hydrogen bands, K. 20 fine lines.
127	" 5	Vega	20	0.50	13 " " K.
128	" 5	Altair	38	0.35	12 " " K.
131	" 6	Regulus	40	0.35	10 " " K. (Night very hazy.)
132	" 6	α Cygni	40	0.20	16 " " K. 10 other lines.
133	" 6	Capella	40	0.40	F, G, h, H, K. 125 lines between F and H.
134	" 7	Jupiter	20	0.80	F, G, h, H, K. Many lines.
135	" 8	Sirius	20	0.35	14 Hydrogen bands, K.
136	" 8	Spica	40	0. 4	12 " " K. 10 other lines.
137	" 8	Jupiter	30	1. 6	F, G, h, H, K. Many lines.
138	" 8	Pollux	60	0. 3	F, G, h, H, K. Many lines.
141	" 12	Jupiter	25	1. 0	Good definition.
142	" 12	"	14	0. 5	" "
147	" 20	"	14	1. 2	Excellent definition.
148	" 20	Vega	24	0. 6	11 Hydrogen bands, K. 5 other lines (hazy).
		Spica	37	0. 4	11 " " K. 10 other lines (hazy).
149	" 20	Procyon	49	0. 4	11 " " K. 75 other lines.
		Sirius	20	0. 8	11 " " K. 10 other lines (Sirius low).
150	" 21	Jupiter	15	1. 5	Excellent definition.

Some of the above plates were measured on the dividing-engine, and it was found that relative wave-lengths could be measured accurately to 0.1 Ångström unit.

The photographs were enlarged (see frontispiece) by a method due to Professor Wadsworth. The plate to be enlarged and widened is attached to a heavy pendulum on an arm near the point of support. Allowing the pendulum to swing, the plate oscillates nearly vertically in front of the camera used for enlarging. Adjusting the plate so that the lines of the spectrum are vertical, we get a spectrum enlarged, and widened to any convenient width. The resulting plate is a little denser on the edges, but this may be cut off in printing.

MOTION IN LINE OF SIGHT.

As has been stated, the telescope to which the grating is attached is used merely as a finder for following the star. The star is allowed to trail in right ascension, in order to widen out the spectrum, but is kept from moving in declination, by using the filar micrometer, and adjusting by means of the slow-motion screws. The object glass has a diameter of $9\frac{1}{2}^{\text{in}}$ with a focal length of 142^{in}. The star can be kept from moving in declination at least to within one second of arc. A shift of $1''$ in declination corresponds for the grating used to 0.01 Ångström unit, which for the dispersion of this grating is a smaller quantity than can be measured. We can thus take the spectrum of any star, and then by pointing at any other star in any part of the heavens, its spectrum may be taken above or below the other, merely by displacing the star, as seen in the filar micrometer, a little in right ascension, keeping the same declination as before. On developing the plate we have the two spectra side by side, and hence we can readily compare the shifts of the different lines.

Neglecting the small shift due to pressure, the shift may be made up of two parts, a true shift due to the differences in the motions of the two stars in the line of sight, and an instrumental shift due to the flexure of the spectroscope and telescope. The shift due to the differences of motion in the line of sight of the two stars varies with the wave-length, the shift due to flexure is the same throughout. But since the spectrum is normal, these two shifts can be separated, the necessary equations being few and simple.

Thus we obtain the relative motion of two stars in the line of sight, and if the motion of one is known, we thus know that of the other. Different stars might be compared with a star whose motion is known as Jupiter, and we would thus obtain the absolute motions in the line of sight, and hence absolute wavelengths.

Following this idea, several combinations of spectra of two stars were photographed. Some of them showed a shift, but

owing to the position of the Observatory, the definition of the photographic plates was not sufficient to allow the plates to be measured with an accuracy necessary for investigating motion in the line of sight. Greater dispersion than that given by the grating used would also increase the accuracy of the measurements.

ABSOLUTE WAVE-LENGTHS.

Absolute wave-lengths can also be obtained by using the grating so as to photograph two order of spectra on the same plate, and by measuring the position of the same line in the two different spectra.

This plan was tried by photographing the first and second order spectra on one plate. Taking advantage of the fact that the second spectrum is normal, the reductions necessary for finding the absolute wave-lengths are very simple. But the dispersion, and the definition obtained from the grating used, and the position of the Observatory, were not sufficiently favorable to permit of any accurate determinations of absolute wave-lengths.

CONCLUSIONS.

In conclusion, the direct concave-grating spectroscope possesses the following advantages:

1. Remarkable simplicity of the apparatus.

2. The ease with which a star can be accurately followed.

3. The spectrum is readily widened by regulating the driving-clock of the equatorial.

4. Great extent of the ultra violet light.

5. Excellent definition.

6. The exposure necessary compares very favorably with other forms of spectroscopes.

7. The spectrum is "normal."

8. Relative motion of two stars may be determined.

9. Absolute wave-lengths, and absolute motion in the line of sight may be determined.

My thanks are especially due to Dr. Charles Lane Poor for the continued kindness and interest he has shown in my work, being at all times and on all occasions ready to advise and assist me. I wish to thank Professor Rowland for the interest shown, and for his kindness in ruling a large grating; also to thank Dr. Ames for his interest and advice; and also to thank Mr. L. E. Jewell who, from his great practical knowledge of spectroscopy and photography, and from his readiness at all times to help, has rendered me valuable assistance.

Samuel Alfred Mitchell, fourth son of John C. and Sarah Mitchell, was born in Kingston, Ontario, April 29, 1874. After obtaining his preliminary education in the public and high schools at Kingston, he entered Queen's University (Kingston) in October 1890. In 1894, he graduated from this institution as Master of Arts, and medallist in mathematics. For a year after graduation he remained at Queen's, holding the position of laboratory assistant in physics. Coming to Johns Hopkins in 1895, he took up studies in astronomy, physics and mathematics. In 1896–7 he was an assistant in the Observatory, and in 1897–8 he held a fellowship in astronomy.